The Chalkboard Activity Structure as a Facilitator of Helping, Understanding, Discussing, and Reflecting

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Abstract

This article brings light to a traditional classroom activity that can be described as having many learning and facilitating qualities. The chalkboard provides an interactive classroom environment where the learning needs of both the teacher and the student can be met. The chalkboard solve-then-explain structure and the chalkboard step-by-step structure are introduced as having the abilities to monitor student performance and provide feedback (giving and receiving). These attributes of *Children's Math Worlds* can be done within various types of cooperative learning groups.
This paper describes results from a 6-year developmental action research project that began in urban Latino classrooms in English and in Spanish. The project was later extended to urban classrooms with children of varied ethnicities and to suburban classrooms. We employed a Vygotskian emergent research model in which analytical theoretical work, design of teaching-learning activities, and empirical work in classrooms was continually intertwined. The teaching/learning activities were based on emergent theories and models of developmental trajectories in children's mathematical thinking and on models of teaching that describe classroom learning trajectories based on the models of children's thinking.

The teaching-learning activities were developed into a curriculum for grades 1, 2, and 3 called Children's Math Worlds (CMW). This curriculum takes a multi-level Piagetian and Vygotskian view of learning-teaching. The analysis of what is being learned by an individual uses a Piagetian perspective of learning as using and building individual conceptual structures for functioning in a domain. The analysis of how mathematical knowledge and skills are learned takes a Vygotskian perspective on learning in mathematics as involving cultural tools of language, written symbols, and cultural solution methods whose cultural meanings must be learned by individuals. The analysis of teaching uses a Vygotskian perspective of teaching as assisting children to construct increasingly more powerful mathematical conceptual structures (see Tharp & Gallimore, 1988, for a fuller explication in non-mathematical areas). This assistance takes many forms including building and orchestrating classroom social structures that assist children and the teacher to facilitate each other's emerging conceptual concepts of mathematics, of the social structures, of each other as individuals, and of each emergent self within the classroom context.

The classroom teaching-learning approaches in CMW link mathematical activities in the classroom to children's mathematical experiences outside of school. They are designed to enable teachers to listen to various cultural and personal expressions of children and to weave these into mathematical activities that affirm children's varying cultural backgrounds while extending everyone's mathematical knowledge using standard mathematical language and written symbols. Children's Math Worlds uses homework every day. It helps teachers mobilize home support for children's mathematical learning by identifying in the home someone to help with mathematics homework; support has been available in most homes. The activities are designed to enable children to solve a range of word problems, to move through learning trajectories from single-digit direct modeling methods to counting-on and counting-up methods and then to ten-structured or known-fact methods, to construct robust multiunit conceptions that enable
children to use multidigit addition, subtraction, multiplication, and division calculation methods involving units of ten, hundred, and thousand, and to construct a conceptual field that organizes interrelated understandings of multiplication, division, fractions, ratio, probability, area, combinations, and rate.

The teaching-learning activities are based on models of children’s conceptual structures: a developmental sequence of finger, drawn, and mental single-digit addition and subtraction methods (Fuson, 1992a, 1992b; Fuson, Perry, & Kwon, 1994; Fuson, Perry, & Ron, 1996), a model of a developmental sequence of multiplication methods (Hufferd-Ackles, 1998), a model of developmental sequences in the solving of word problems (Fuson, 1994), and a model of a connected web of multidigit conceptual structures that are usable in several different mental, drawn, and numeric multidigit addition and subtraction methods (Fuson & Smith, 1995, in press; Fuson, Smith, & Lo Cicero, 1997; Fuson et al., 1997). The teaching approaches use an explicit model of teaching word problem solving in an algebraic and linguistic manner (Fuson, Hudson, & Ron, in press; Ron, in press), partially explicated (Fuson & Smith, in press) or implicit models of teaching in the other domains, and an equity pedagogy that applies across the mathematical domains (Fuson et al., 1998).

Though over 90% of the children in CMW urban classrooms meet Federal guidelines for the free-lunch program, CMW children outperform U.S. children from a range of backgrounds who receive traditional mathematics instruction, outperform on many tasks children from a range of backgrounds who use the reform math Everyday Mathematics program, outperform on many tasks Chinese children, and, on some tasks, equal or exceed the performance of Japanese children (Fuson, Smith, & Lo Cicero, 1996; Fuson, 1996). Suburban CMW children do even better than urban CMW children on some tasks (and thus outperform the other samples considerably), but on many tasks the urban-suburban gap is not as wide in the two CMW groups as is characteristic of such comparisons (Fuson, 1998).

This paper focuses particularly on classroom social structures that can support the meaningful learning of mathematics by urban and suburban children. We first overview the general macrostructure of a daily CMW class. We then describe two classroom participant structures--the chalkboard solve-then-explain structure and the chalkboard step-by-step structure--that we have found to facilitate learning in our various mathematical domains and that have been possible for a range of teachers to use in urban and suburban schools.

Overview of a CMW Class Period

Figure 1 shows an overview of the parts of a CMW teaching day. The major part of a class period is focused on developing concepts and strengthening knowledge. This is accomplished by using a mixture of whole-class co-constructed instructional conversations (Goldenberg, 1992/3; Tharp & Gallimore, 1988) and individual work carried out with various kinds of
available supports. Days vary in the balance and patterns of interaction of these two types of participant structures. These whole-class instructional conversations and focused individual work form reflective cycles in which (a) whole-class demonstrating and explaining can help individuals to build stronger understandings and (b) individual progress in understandings can be reported to the whole class to further understandings both of the reporting student and of the rest of the class.

An instructional conversation differs from the traditional IRE structure of classrooms (Cazden, 1988) in being more adapted to knowledge and input of participants, in facilitating feeling of involvement and belonging, and in focusing heavily on meaning-making by all participants. The teacher on a given day has both general and specific instructional goals, but the path for achieving these goals often arises in an emergent fashion from the particular solution methods, errors, or word problems posed by children. Several aspects of an instructional conversation play especially important roles in CMW classrooms. Mathematical and ordinary language are strengthened by having children restate problems in their own words, generate as well as solve problems, explain how they solved a problem, and describe mathematical experiences they have outside the classroom. Issues and errors are clarified by questions raised or clarification given by children or the teacher about the work or explanations of other children or of the teacher and by the teacher raising issues or errors observed in the homework or classwork. Mathematizing a situation is an important way to link children’s lives to mathematical concepts; to mathematize, a teacher starts with some familiar situation or story described by a child and focuses in on mathematical attributes of the situation (Lo Cicero, Fuson, & Allexsaht-Snider, in press). Mathematical modeling of a situation or word problem by making a math drawing is a pervasive attribute of CMW activities (Fuson et al, 1998). Such meaningfully drawn models facilitate communication about and reflection on the mathematical thinking and solution method of the child making the drawing; unlike object manipulative materials, they also are available for the teacher to consider after class is over. Sharing solution methods, new insights about some problem or situation, and examples of mathematical concepts serves both mathematical learning goals and social goals, enabling children to participate in experiences or thinking of other children.

Most individual CMW work is carried out in contexts designed to make help available when it is needed. Because one teacher cannot simultaneously help many students, we have developed several different kinds of peer-helping structures. Explicit helping pairs are used for some tasks in which at least half of the class can do and understands a given crucial task. Although we find some natural helpers in every class including first grade, most children require modeling, discussing, and evaluating of helping practices in order to move from “doing for” or “telling how” to more conceptually-based helping methods such as asking questions, monitoring and
helping only with some part of the task, encouraging, giving hints, and explaining why. Informal seeking of help as needed from a near-by peer often works well as an activity structure, but it only helps those who feel in need of help.

A more complex constant need in mathematics classrooms is to monitor and give feedback concerning the correctness of work, to intervene with help before some incorrect notion becomes too fixed, and to provide sufficient venues for children to practice describing and explaining their mathematical thinking. Doing this continually for 20 or more children is a daunting task. Our project has made some progress in designing such monitoring and feedback, but we in collaboration with teachers still are searching for more ways to address these issues. Homogeneous helping pairs, heterogeneous helping pairs, homogeneous working pairs, and similar compositions of small groups are each useful in some situations. But each requires time for children to learn productive social norms in these situations. Our use of meaningfully drawn models in many areas does mean that the teacher at the end of a day or when looking at homework does have a trace of mathematical thinking. But such thinking might have been done with assistance in class or at home by the home helper.

Two participant structures for children working at the chalkboard provide excellent opportunities for monitored performance and the giving and receiving of feedback on complex multi-step processes. These participant structures--the chalkboard solve-then-explain structure and the chalkboard step-by-step structure--cycle between the whole-class and individual work structures with varied amounts of monitoring of performance and feedback giving/receiving. Each of these is discussed more fully below.

Other attributes of a CMW class period are in the vertical pieces of Figure 1. Testing for instructional purposes occurs on some days at the beginning of the period. In most other CMW participant structures, help for individuals is available from someone. However, the test/quiz participant structure emphasizes unassisted performance to give feedback to each individual about the extent to which help and more work is needed on some kinds of mathematical tasks. Tests and quizzes are given at the beginning of the period to simulate the primary conditions under which mathematical knowings are used: not just within a few minutes of a review or practice. Short quizzes for instructional purposes are used frequently to indicate kinds of errors children are making so that these can be addressed immediately. Spontaneous helping in most other CMW classroom structures makes it more difficult for the teacher, and even for the children themselves, to judge when a given child can do some kind of mathematical thinking without assistance.

Teachers use different methods of scanning homework for frequent errors and of monitoring homework completion. Homework may or may not be discussed on a given day, depending on whether mathematical or emotional/motivational issues need to be addressed.
Coherence over the year can be facilitated by the use of re-viewing methods before and after a lesson. These methods are common in Japanese classrooms (Stevenson & Stigler, 1992). These re-viewings are brief summaries by a child or the teacher of what was learned yesterday and by a child or teacher re-view at the end of the math period of important central mathematical issues. This helps place all children within a learning stream in which the fact that they are learning, as well as what they are learning, is made salient and can facilitate awareness of and reflection about the learning process. This common class history can also give all participants a sense of meaningful progress.

Practice plays important roles in the CMW curriculum. In order for the less advanced children to move through a developmental trajectory of more advanced methods, they need to learn and understand certain knowledge skills and then achieve fluidity with them so that they can be used in more complex tasks. For example, a substantial number of urban second graders begin the year without being able to count to 100 by tens or by ones. If these children are to carry out solution methods for adding and subtracting 2-digit numbers that operate on groups of ten, they must learn and understand tens groupings and be able to count them and combine them with groups of ones. If such crucial knowledge skills are not learned and practiced, these children must use slow and inaccurate methods of drawing and counting only by ones. We view it as entailed upon a curriculum to organize initial learning activities and then provide sufficient practice of such crucial knowledge skills. Practice is in the first column of Figure 1 and is labeled “getting faster.” It always follows initial development of the concept involved in the practice. After such conceptual development, practice may occur within the regular class or in a special practice time such as in the beginning of the day or at the end of the day. The latter two times have the benefit of turning what is often “down time” into effective learning time. Various kinds of practice routines are used, including individual, pair, and whole-class practice. Regularly completing homework also is an important component of CMW practice. Homework is usually separated into one sheet doing new work and one sheet practicing knowledge skills introduced earlier in the year or in the unit.

The Chalkboard Solve-Then-Explain Structure

A major component of the CMW Project has been to develop in collaboration with teachers classroom participant structures that maximize learning time and facilitate reform learning as well as traditional learning goals. A traditional activity structure—sending several children to work at the board while other children work at their seats—has proved to be a very powerful facilitator of reform kinds of learning. In this structure several children solve a problem at the board while everyone else solves it at their seats. The problem may come from a child, the teacher, or the curriculum. A crucial aspect of our adaptation of this participant structure is that, for most problems, children make a meaningfully drawn model of their problem situation
or solution method. With larger numbers, they may also show their numerical work. These drawings and numerical work communicate the solver’s thinking to other children and to the teacher. The teacher can see the thinking of the children at the board and select methods s/he wishes to highlight in the subsequent discussion.

Next, two or three of the children at the board describe or explain in turn how they solved the problem. To clarify these descriptions/explanations, classmates or the teacher may then ask questions of the explainer or make clarifying comments. The visible meaningfully drawn models serve as a referent for this class discussion, making that discussion more accessible to all of the listeners in the classroom. The presence of multiple methods on the board facilitates comparisons as well as underscoring the fact that multiple methods are desirable.

Finally, the solve-then-explain cycle is repeated as several more children go to the board to solve the next problem. The teacher can cycle through the whole class in one or two class periods, thus assessing everyone’s thinking.

This participant structure also has other advantages. First, no time is wasted while children draw their method on the board; everyone is working during that time. A related typical reform method lets children do problems at their seat and then the teacher selects a child or children to send to the board. Children at their seats often do nothing while the methods are being written on the board, thus wasting valuable class learning time. Second, spontaneous helping occurs very frequently by children adjacent to each other at the board. The shared space and easily visible work seem to elicit such helping. This helping sometimes is solicited by the child needing it and sometimes is offered by the adjacent child who is monitoring the work of their adjacent peer. Third, children love to go to the board, so this structure is motivating to most children. Some predictable fair way to take turns going to the board (e.g., each row or table in turn) facilitates both children’s faith in the equity of their classroom and speeds up the transition when new children are going to the board. Children do not seem as concerned about getting a chance to describe their thinking, so teachers have considerable flexibility to pursue particular kinds of mathematical thinking as well as to give, over time, all children practice and support in improving their explaining of their thinking.

**The Step-By-Step Chalkboard Process:**

**Consolidating and Understanding Complex Multiple-Step Mathematical Solution Methods**

A variation of the solve-and-explain method is particularly useful for problems that have many subcomponents (e.g., 3-digit subtraction with borrowing). Such problems are difficult for less advanced children, who initially need help in coordinating all of the subcomponents. Multidigit computation with regrouping makes both heavy procedural and conceptual demands upon children. To do it accurately, they must maintain the different quantity meanings (ones, tens, hundreds) of several digits across different interlinked representations (number words,
number digits, drawn quantity representations) while they carry out several different steps in a solution method. This must occur in the face of the appearance of the numbers that lures children into seeing and treating multi-digit numbers as isolated single digits (they do all look the same). Even after a child can carry out a method successfully, s/he may not be able to describe the method very well. The existence of the several subcomponents makes describing one's solution method a complex task.

Even in the classrooms most successful at establishing understanding, accuracy, and the social norm of describing one’s method, there is a significant gap in time between when the most-advanced and the least-advanced students understand what they are doing (and also between when the different students can describe what they are doing). The step-by-step chalkboard structure addresses this gap. It is begun after children have had opportunities to invent methods and establish meanings for multidigit operations. It is a consolidation step, mainly for the benefit of the least-advanced quarter or so of the class who need help to systematically, consistently, and accurately put it all together. This participant structure is essentially a way to make the rich feedback and interactive explanation achievable in tutoring contexts available to many children at once.

The step-by-step chalkboard structure functions as follows:

(a) Four or five children at the board work on the same problem, as does the rest of the class at their seats. Each child will explain and do only one step of the problem.

(b) A child describes the step s/he will do next before doing it. Everyone else at the board and at their seats then carries out that step.

(c) If a child has difficulties with a step, that step can be discussed again by another child or the teacher, after it is visible on the board. This step-by-step focus enables the teacher to focus discussion or move on as needed.

(d) Then the next child describes what step will be done next and everyone does it.

The teacher may direct attention to particular methods by putting children typically using those methods at key steps. In this way the class may benefit by exploring a more advanced method some child has invented or may focus on a safe, accessible fall-back method to help the least advanced children. The step-by-step nature allows children to carry out an unfamiliar method more easily than trying to do the whole method at once. Thus, children can “get inside” other methods in order to try to understand them better.

The central advantages of this structure are the following. First, children learn to connect their actions to reasons for them and can practice giving oral descriptions and explanations. Rather than merely carrying out a procedure, they are collaboratively weaving a meaningful
narrative that firmly establishes a memorable framework of understanding across the entire multistep activity.

Second, the structure maximizes the chances that the least advanced children in the class will be successful because (a) they are only responsible for describing and then doing alone one step and (b) their step is preceded by correctly modeled, detailed, and explained steps (and followed by these as well).

Third, error discussion and correction is not a dragged-out painful affair. A child is only responsible for one step and does not face getting lost in a series of mistakes across an entire problem. Also, children at the board tend to be highly proactive. If they are unsure about what comes next, they tend to ask each other before their turn comes up. If they make a mistake, another child at the board often quickly leans over to help correct or chips in during the explanation to make it more complete or correct.

Fourth, the teacher (or other children at their seats) can ask questions to clarify or emphasize certain aspects of explanations or direct attention to meanings that will correct mistakes or clarify why that step is done in that way. The elaborated, slowed-down, step-by-step focus permits any intervention at each step to be precise, focused, and adapted to needs.

In sum, use of the step-by-step method follows a period of exploration in which students have devised or chosen their own method. Its major functions are to support focusing in on particular difficult steps of a multistep method so that these steps can be understood and done correctly and can be explained. This structure allows both a high degree of individually-generated explanation, yet also detailed rich modeling of steps by other children for those who need it. It permits whole-class activity and detailed, meaning-focused tutoring with feedback at the same time. More advanced students can be placed in key positions, initially in order to model full, meaningful descriptions and clear, accurate steps. Later on, less-advanced students are placed at those positions so that the teacher can monitor their understanding and so that they have an opportunity to describe and justify the more complex steps. Spontaneous helping may occur as a turn approaches if a given student is not sure of his/her step.

**Conclusion**

New educational goals may require new kinds of classroom participant structures. Because each participant structure requires time for children to learn how to participate effectively, designing a few powerful participant structures that are useful across a range of mathematical topics is an important goal for reform programs. We have presented here a description of the major elements of a CMW class period and two special chalkboard participant structures developed for central frequent needs. These chalkboard structures indicate that the traditional classroom structure of children working at the board can facilitate a focus on alternative methods, children's descriptions of their thinking, and understanding and sense-making for all.
They also can facilitate peer and teacher helping of less advanced students. The step-by-step method enables everyone to try out a given method. Such experiencing goes beyond merely listening to another describe that method. It also scaffolds less advanced students, both in problem solving and in explaining their work.
References


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<td><strong>Practice on pieces already conceptually understood to enable their use in further conceptual development</strong></td>
<td><strong>Whole-Class Co-Constructed Instructional Conversation</strong></td>
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<tr>
<td>Finger practice, mental math, count-by-x, counting money</td>
<td>The instructional conversation focuses heavily on meaning-making by all participants and thus attempts to adapt to knowledge and input of individuals. It stimulates feelings of involvement and belonging. Each individual interprets the conversation according to his/her own knowings.</td>
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<td><strong>Individual self-regulated and goal-directed practice</strong></td>
<td><strong>Individual conceptual development through teacher and student demonstrating, explaining, problem posing, and problem solving</strong></td>
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<td>Giving self problems, using needed flashcards or study sheets</td>
<td>Building language; clarifying issues and errors; mathematizing a situation; math modeling using drawings; sharing solution methods, new insights, and examples of mathematical concepts</td>
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<tr>
<td><strong>Testing for Instructional Purposes (as needed): Working alone before any review</strong></td>
<td><strong>Chalkboard Solve-Then-Explain Structure</strong>: 4 or 5 students at the board and others at seats solve a whole problem, then 2 or 3 students at the board explain their method; peer or teacher question and/or clarify; compare methods</td>
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<td><strong>Chalkboard Step-By-Step Structure (consolidation and focus on key steps)</strong>: Student tells a step, everyone does it (4 or 5 at board, others at seats), explain and discuss step if necessary; a different student tells next step; etc.</td>
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**Figure 1: Children's Math Worlds Teaching Day Overview**